From Annihilation to Horizons: A Finite Algebraic Toy Model for Reactive Matter, Radiation, and Gravitational Collapse

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July 26, 2026

Abstract

Matter–antimatter annihilation, radiative transport, and gravitational collapse are individually established components of mathematical physics. Their full description ordinarily requires relativistic kinetic equations coupled to Einstein's field equations. This paper introduces a finite algebraic toy model that compresses these components into an auditable state-transition system. The local state consists of matter M, antimatter $\bar{M}$, radiation R, and geometry g. Matter and antimatter annihilate into radiation while conserving total energy. The accumulated stress-energy then determines whether the system remains dispersive or enters a trapped regime. Crucially, a black hole is not represented as a new material substance. Horizon formation is treated as a geometric transition induced by concentration of the conserved energy content. The model therefore distinguishes three processes: opposition produces radiation, transport redistributes energy, and concentration alters causal accessibility. The construction is not proposed as a replacement for Einstein–Boltzmann dynamics. Its purpose is to isolate their logical architecture in a minimal finite framework suitable for exhaustive computation, verification, and comparison with continuum theory. The resulting algebra provides a candidate bridge between reaction networks, discrete dynamical systems, causal geometry, and gravitational critical phenomena.

Keywords: matter–antimatter annihilation; relativistic kinetic theory; gravitational collapse; event horizon; finite dynamical system; reaction algebra; stress-energy; toy model.

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1. Scope

The established continuum description has the form

$$p^{\mu}\nabla_{\mu}f_s = C_s[f], \qquad (1)$$

for matter, antimatter, and radiation distributions f_s, coupled to

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}. \qquad (2)$$

The present construction does not replace (1)–(2). It seeks their smallest finite logical analogue.

The intended compression is

$$\boxed{\text{reaction} \;+\; \text{transport} \;+\; \text{concentration} \;+\; \text{causal transition.}} \qquad (3)$$

The model must preserve four distinctions:

$$\begin{array}{rl} \text{annihilation} & \neq \text{deletion}, \\ \text{radiation} & \neq \text{absence}, \\ \text{high energy} & \neq \text{horizon}, \\ \text{horizon} & \neq \text{new matter species}. \end{array} \qquad (4)$$

2. Finite State Space

Let Xbe a finite set of spatial cells. At each $x \in X$and discrete time $t \in \mathbb{N}$, define

$$S_t(x) = (m_t(x), \bar{m}_t(x), r_t(x), g_t(x)). \qquad (5)$$

Here

$$m_t(x), \bar{m}_t(x), r_t(x) \in \mathbb{N} \qquad (6)$$

denote matter, antimatter, and radiation energy units.

The geometric variable is

$$g_t(x) \in \{0,1\}, \qquad (7)$$

with interpretation

$$g_t(x) = \begin{cases} 0, & \text{untrapped cell,} \\ 1, & \text{trapped cell.} \end{cases} \qquad (8)$$

The complete configuration space is

$$\mathcal{S} = \prod_{x \in X} (\mathbb{N}^3 \times \{0,1\}). \qquad (9)$$

For bounded computation, one may replace $\mathbb{N}$ by

$$E_N = \{0,1, \dots, N\}. \qquad (10)$$

Definition 1. Visible and geometric variables

The material-radiative content is

$$q_t(x) = (m_t(x), \bar{m}_t(x), r_t(x)). \qquad (11)$$

The causal-accessibility state is $g_t(x)$.

Thus

$$S_t(x) = (q_t(x), g_t(x)). \qquad (12)$$

This separation prevents the identification

$$\text{black hole} = \text{black-hole matter}. \qquad (13)$$

Instead,

$$\boxed{\text{matter and radiation source geometry;geometry determines accessibility.}} \qquad (14)$$

3. Local Annihilation Algebra

At each cell define the annihilating amount

$$a_t(x) = \min\{m_t(x), \bar{m}_t(x)\}. \qquad (15)$$

The local reaction map A is

$$A\colon (m, \bar{m}, r) \mapsto (m - a, \bar{m} - a, r + 2a), \qquad (16)$$

where

$$a = \min\{m, \bar{m}\}. \qquad (17)$$

This is the finite analogue of

$$M + \bar{M} \longrightarrow R. \qquad (18)$$

Proposition 1. Local energy conservation

Define the local energy count

$$E(m, \bar{m}, r) = m + \bar{m} + r. \qquad (19)$$

Then

$$E(A(m, \bar{m}, r)) = E(m, \bar{m}, r). \qquad (20)$$

Proof

By (16),

$$\begin{aligned} E' &= (m - a) + (\bar{m} - a) + (r + 2a) \\ &= m + \bar{m} + r. \end{aligned} \qquad (21)$$

Hence $E' = E$. ■

Corollary 1. Complete local annihilation

If

$$m = \bar{m}, \qquad (22)$$

then

$$A(m, m, r) = (0,0, r + 2m). \qquad (23)$$

Equal matter and antimatter therefore produce radiation, not zero:

$$\boxed{(m, m, 0) \longmapsto (0,0,2m).} \qquad (24)$$

4. Transport on a Finite Causal Graph

Let

$$\Gamma = (X, \mathcal{E}) \qquad (25)$$

be a finite directed graph. An edge

$$x \to y \qquad (26)$$

means that energy may propagate from cell $x$to cell $y$during one time step.

For each species

$$s \in \{m, \bar{m}, r\}, \qquad (27)$$

let

$$P_t^s(x, y) \in [0,1] \qquad (28)$$

be the transported fraction from x to y.

For an untrapped cell, require

$$\sum_{y \in X} P_t^s\,(x, y) = 1. \qquad (29)$$

The transport operator is

$$(T_t^s q)(y) = \sum_{x \in X} P_t^s\,(x, y) q(x). \qquad (30)$$

For integer-valued implementations, the coefficients may be replaced by exact routing counts.

Definition 2. Geometrically restricted transport

Let the accessibility function be

$$\chi_t(x,y) = \begin{cases} 1, & \text{transport from } x \text{ to } y \text{ is causally permitted,} \\ 0, & \text{otherwise.} \end{cases} \qquad (31)$$

Then

$$P_t^S(x,y) = 0 \text{whenever } \chi_t(x,y) = 0. \qquad (32)$$

The geometry therefore constrains transport through the support condition

$$\operatorname{supp} P_t^S \subseteq \operatorname{supp} \chi_t. \qquad (33)$$

Definition 3. Horizon condition

A trapped cell satisfies

$$g_t(x) = 1 \Longrightarrow \chi_t(x,y) = 0 \text{for all exterior } y. \qquad (34)$$

Inward transport may remain permitted:

$$\chi_t(y,x) = 1 \qquad (35)$$

for selected exterior cells y.

Thus the finite horizon condition is asymmetric:

$$\boxed{\text{entry may occur;escape does not.}} \qquad (36)$$

Proposition 2. Global conservation under transport

Suppose

$$\sum_{y \in X} P_t^S\,(x,y) = 1 \qquad (37)$$

for every x and species s. Then

$$\sum_{y\in X}(T_t^s q)(y) = \sum_{x\in X} q(x). \qquad (38)$$

Proof

Using (30),

$$\begin{aligned}\sum_y (T_t^s q)(y) &= \sum_y \sum_x P_t^s(x,y) q(x) \\ &= \sum_x q(x) \sum_y P_t^s(x,y) \qquad (39) \\ &= \sum_x q(x).\end{aligned}$$

Therefore transport redistributes the species without changing its total quantity. ■

5. Reaction–Transport Evolution

Let

$$A_X = \prod_{x\in X} A_x \qquad (40)$$

denote simultaneous local annihilation over all cells.

Let

$$T_t = T_t^m \times T_t^{\tilde{m}} \times T_t^r \qquad (41)$$

denote species transport.

The pre-geometric update is

$$q_{t+1}^{-} = T_t A_X(q_t). \qquad (42)$$

The superscript "−" indicates that geometry has not yet been updated.

The local post-transport energy is

$$e^{-}_{t+1}(x) = m^{-}_{t+1}(x) + \bar{m}^{-}_{t+1}(x) + r^{-}_{t+1}(x). \qquad (43)$$

Proposition 3. Global energy conservation before trapping

Define

$$E_t = \sum_{x \in X} (m_t(x) + \bar{m}_t(x) + r_t(x)) . \qquad (44)$$

If the reaction map is given by (16) and transport satisfies (37), then

$$E^{-}_{t+1} = E_t. \qquad (45)$$

Proof

The local reaction conserves energy by Proposition 1. Transport conserves each species by Proposition 2. Their composition therefore conserves the sum. ■

The ordering

$$q_t \overset{A_X}{\rightarrow} \tilde{q}_t \overset{T_t}{\rightarrow} q^{-}_{t+1} \qquad (46)$$

is not mandatory. One may instead transport before reaction:

$$q_t \overset{T_t}{\rightarrow} \hat{q}_t \overset{A_X}{\rightarrow} q^{-}_{t+1}. \qquad (47)$$

These two updates generally differ:

$$T_t A_X \neq A_X T_t. \qquad (48)$$

Equation (48) is the finite analogue of reaction–transport noncommutativity.

Definition 4. Operator splitting defect

Define

$$\Delta_t(q) = T_t A_X(q) - A_X T_t(q). \qquad (49)$$

Then

$$\Delta_t(q) = 0 \qquad (50)$$

exactly when local annihilation and transport commute on q.

The magnitude

$$\| \Delta_t(q) \| \qquad (51)$$

measures sensitivity to the chosen discrete update order.

6. Discrete Concentration Functional

Gravitational collapse depends not only on total energy but on its spatial concentration.

Let

$$B_\rho(x) \subseteq X \qquad (52)$$

be the graph ball of radius ρcentered at x.

Define the enclosed energy

$$E_t(x, \rho) = \sum_{y \in B_\rho(x)} e_t\,(y). \qquad (53)$$

Let

$$R(x, \rho) > 0 \qquad (54)$$

be an assigned effective geometric radius for that ball.

The finite compactness functional is

$$\mathcal{C}_t(x, \rho) = \frac{2GE_t(x, \rho)}{c^4 R(x, \rho)}. \qquad (55)$$

In units

$$G = c = 1, \qquad (56)$$

this becomes

$$\mathcal{C}_t(x,\rho) = \frac{2E_t(x,\rho)}{R(x,\rho)}. \qquad (57)$$

Equation (55) is motivated by the Schwarzschild compactness ratio

$$\frac{2GM}{c^2R}, \qquad (58)$$

after interpreting $E = Mc^2$.

Definition 5. Critical concentration

Fix a threshold

$$\mathcal{C}_* > 0. \qquad (59)$$

A region is supercritical when

$$\mathcal{C}_t(x,\rho) \geq \mathcal{C}_*. \qquad (60)$$

Define the maximum local compactness

$$\mathcal{C}_t^{\max}(x) = \max_{\rho\in\mathcal{R}} \mathcal{C}_t(x,\rho), \qquad (61)$$

where $\mathcal{R}$is the finite set of admitted radii.

The geometric update candidate is

$$\hat{g}_{t+1}(x) = \mathbf{1}[\mathcal{C}_{t+1}^{\max}(x) \geq \mathcal{C}_*]. \qquad (62)$$

Remark 1. Threshold status

Equation (62) is not a derivation of an event horizon from Einstein's equations. It is a coarse finite surrogate for the onset of trapping.

Its purpose is to distinguish

large energy (63)

from

large energy concentrated within a small effective radius. (64)

Accordingly,

$$E_t \gg 1 \qquad (65)$$

does not imply trapping unless

$$\mathcal{C}_t^{\max} \geq \mathcal{C}_*. \qquad (66)$$

Definition 6. Persistent trapping

To prevent a trapped cell from becoming untrapped by a single subsequent fluctuation, define

$$g_{t+1}(x) = g_t(x) \vee \hat{g}_{t+1}(x). \qquad (67)$$

Equivalently,

$$g_{t+1}(x) = \max\{g_t(x), \hat{g}_{t+1}(x)\}. \qquad (68)$$

Thus

$$g_t(x) = 1 \Longrightarrow g_{t+1}(x) = 1. \qquad (69)$$

The trapped set

$$H_t = \{x \in X : g_t(x) = 1\} \qquad (70)$$

is therefore monotone:

$$H_t \subseteq H_{t+1}. \qquad (71)$$

Proposition 4. Finite stabilization of the trapped set

If $X$is finite and trapping is persistent, then there exists

$$t_* \leq | X | \qquad (72)$$

after the final strict growth event such that

$$H_t = H_{t_*} \text{for all } t \geq t_*. \qquad (73)$$

Proof

Each strict inclusion

$$H_t \subsetneq H_{t+1} \qquad (74)$$

adds at least one new cell. Since Xcontains only | X |cells, at most | X |strict inclusions can occur. ■

7. Exterior and Interior Energy Accounting

Although a horizon is not a material species, it is useful to distinguish energy by causal accessibility.

Define the exterior set

$$X_t^{\text{ext}} = X \setminus H_t. \qquad (75)$$

The exterior-accessible energy is

$$E_t^{\text{ext}} = \sum_{x \in X_t^{\text{ext}}} e_t(x). \qquad (76)$$

The trapped energy is

$$E_t^{\text{trap}} = \sum_{x \in H_t} e_t(x). \qquad (77)$$

Hence

$$E_t = E_t^{\text{ext}} + E_t^{\text{trap}}. \qquad (78)$$

The quantity E_t^{trap} is not a fourth substance. It is the energy of the original matter, antimatter, and radiation fields restricted to the trapped region.

Thus

$$\boxed{E^{\mathrm{trap}} = \text{ordinary conserved energy classified geometrically.}} \qquad (79)$$

Definition 7. Exterior observation map

Define

$$\mathcal{O}_t : \mathcal{S} \longrightarrow \mathcal{S}_t^{\mathrm{ext}} \qquad (80)$$

by restriction to the exterior:

$$\mathcal{O}_t(S) = S\,|_{X_t^{\mathrm{ext}}}. \qquad (81)$$

Two global states are exterior-equivalent when

$$S \sim_t S' \Leftrightarrow \mathcal{O}_t(S) = \mathcal{O}_t(S'). \qquad (82)$$

The exterior observer therefore sees the quotient

$$\mathcal{S}/\sim_t. \qquad (83)$$

Interior differences may remain physically present while being absent from the exterior description.

Proposition 5. Exterior non-injectivity after trapping

If $H_t \neq \emptyset$, then the observation map $\mathcal{O}_t$ is generally non-injective.

Proof

Choose two states S, S' that agree on X_t^{ext} but differ at some $x \in H_t$. Then

$$S \neq S', \qquad (84)$$

while

$$\mathcal{O}_t(S) = \mathcal{O}_t(S'). \qquad (85)$$

Hence $\mathcal{O}_t$ is not injective. ■

This gives a finite distinction between

global state information (86)

and

exterior-accessible state information. (87)

It does not establish information destruction. It establishes only loss of injectivity under exterior restriction.

8. Horizon Flux

Let

$$J_t^s(x,y) = P_t^s(x,y)q_t^s(x) \qquad (88)$$

be the transported amount of species $s$from $x$to y.

Define the inward horizon flux

$$\Phi_t^{\mathrm{in}} = \sum_s \sum_{\substack{x\in X_t^{\mathrm{ext}} \\ y\in H_t}} J_t^s\,(x,y). \qquad (89)$$

Define the outward horizon flux

$$\Phi_t^{\mathrm{out}} = \sum_s \sum_{\substack{x\in H_t \\ y\in X_t^{\mathrm{ext}}}} J_t^s\,(x,y). \qquad (90)$$

The one-way horizon condition requires

$$\Phi_t^{\mathrm{out}} = 0. \qquad (91)$$

Proposition 6. Exterior energy balance

Assume global transport conservation and no exterior source or sink. Then

$$E_{t+1}^{\mathrm{ext}} - E_t^{\mathrm{ext}} = -\Phi_t^{\mathrm{in}} - \Delta_t^{\mathrm{capture}}, \qquad (92)$$

where

$$\Delta_t^{\text{capture}} = \sum_{x \in H_{t+1} \setminus H_t} e_{t+1}^{-}(x) \qquad (93)$$

is the energy newly reclassified as trapped by horizon growth.

Proof

Exterior energy can decrease in two ways:

1. energy crosses from the exterior into an already trapped cell;
2. an exterior cell becomes trapped.

No energy returns across the horizon by (91). Global energy remains conserved. Therefore the exterior change is the negative sum of inward flux and newly captured energy. ■

Correspondingly,

$$E_{t+1}^{\text{trap}} - E_t^{\text{trap}} = \Phi_t^{\text{in}} + \Delta_t^{\text{capture}}. \qquad (94)$$

Adding (92) and (94) gives

$$E_{t+1} = E_t. \qquad (95)$$

Thus apparent disappearance from the exterior is not deletion:

$$\boxed{\Delta E^{\text{ext}} < 0 \Longrightarrow \Delta E^{\text{trap}} > 0.} \qquad (96)$$

Definition 8. Absorbing trapped region

A trapped set H_t is absorbing when

$$x \in H_t, P_t^S(x, y) > 0 \Longrightarrow y \in H_t. \qquad (97)$$

Equivalently,

$$T_t^S(\operatorname{supp} q \subseteq H_t) \subseteq (\operatorname{supp} T_t^S q \subseteq H_t). \qquad (98)$$

Proposition 7. Monotonicity of trapped energy

If $H_t$is persistent and absorbing, then

$$E_{t+1}^{\mathrm{trap}} \geq E_t^{\mathrm{trap}}. \qquad (99)$$

Proof

No energy leaves H_t, while inward flux and horizon growth may add energy. Therefore (94) is nonnegative. ■

9. The Complete Update Map

The finite evolution consists of four stages:

$$\boxed{\text{annihilation} \rightarrow \text{transport} \rightarrow \text{compactness evaluation} \rightarrow \text{causal update.}} \qquad (100)$$

Let

$$\mathcal{A} \qquad (101)$$

denote local annihilation,

$$\mathcal{T}_g \qquad (102)$$

denote geometry-dependent transport,

$$\mathcal{K} \qquad (103)$$

denote compactness evaluation, and

$$\mathcal{G} \qquad (104)$$

denote geometric trapping.

The full evolution map is

$$F: \mathcal{S} \longrightarrow \mathcal{S}, \qquad (105)$$

with

$$F = \mathcal{G} \circ \mathcal{K} \circ \mathcal{T}_g \circ \mathcal{A}. \qquad (106)$$

Explicitly,

$$S_t = (q_t, g_t) \mapsto (q_{t+1}, g_{t+1}). \qquad (107)$$

The material update is

$$q_{t+1} = \mathcal{T}_{g_t} \mathcal{A}(q_t), \qquad (108)$$

and the geometric update is

$$g_{t+1}(x) = g_t(x) \vee \mathbf{1}\left[\max_{\rho \in \mathcal{R}} \frac{2G E^-_{t+1}(x, \rho)}{c^4 R(x, \rho)} \geq \mathcal{C}_* \right]. \qquad (109)$$

Equation (109) shows that geometry is not independently assigned. It is determined by the post-reaction, post-transport energy distribution.

Definition 9. Closed finite universe

The system is closed when:

$$\sum_y P^s_t\,(x, y) = 1, \qquad (110)$$

all reaction products remain represented in q, and no external source term is added.

Theorem 1. Global conservation under the complete update

For a closed finite universe governed by (106), the total energy

$$E_t = \sum_{x \in X} (m_t(x) + \bar{m}_t(x) + r_t(x)) \qquad (111)$$

satisfies

$$E_{t+1} = E_t. \qquad (112)$$

Proof

The annihilation map conserves energy by Proposition 1. Transport conserves energy by Proposition 2. Compactness evaluation changes no material variable. The geometric update changes only causal accessibility and does not alter $m, \bar{m}, r$. Therefore their composition preserves E_t. ■

Corollary 2. Geometry is energetically neutral

The update

$$g_t \mapsto g_{t+1} \qquad (113)$$

does not itself create or destroy energy:

$$\Delta_g E = 0. \qquad (114)$$

Geometry alters permitted future trajectories rather than the conserved energy count.

10. Matter–Antimatter Imbalance

Define the local signed matter charge

$$Q_t(x) = m_t(x) - \bar{m}_t(x). \qquad (115)$$

The global charge is

$$Q_t = \sum_{x \in X} Q_t\,(x). \qquad (116)$$

Proposition 8. Annihilation preserves matter–antimatter imbalance

Under the local annihilation map,

$$Q' = Q. \qquad (117)$$

Proof

From (16),

$$\begin{aligned} Q' &= (m - a) - (\bar{m} - a) \\ &= m - \bar{m} \\ &= Q. \end{aligned} \qquad (118)$$

■

If matter and antimatter transport obey separately conservative kernels, then

$$Q_{t+1} = Q_t. \qquad (119)$$

Thus the finite theory has two global invariants:

$$\boxed{E_t = E_0, Q_t = Q_0.} \qquad (120)$$

Radiation carries energy but no signed matter imbalance.

Corollary 3. Residual matter

For a single isolated cell initially containing

$$(m_0, \bar{m}_0, r_0), \qquad (121)$$

complete annihilation gives

$$m_\infty = \max\{Q_0, 0\}, \qquad (122)$$
$$\bar{m}_\infty = \max\{-Q_0, 0\}, \qquad (123)$$

and

$$r_\infty = r_0 + 2\min\{m_0, \bar{m}_0\}. \qquad (124)$$

Therefore

$$m_\infty + \bar{m}_\infty = \mid Q_0 \mid. \qquad (125)$$

The surviving material content is exactly the initial matter–antimatter imbalance.

11. Finite Dynamical Consequences

Assume bounded energy levels

$$m_t(x), \bar{m}_t(x), r_t(x) \in E_N = \{0,1, \dots, N\}, \qquad (126)$$

and deterministic reaction, transport, and geometric updates.

Then the state space

$$\mathcal{S}_N = \prod_{x \in X} (E_N^3 \times \{0,1\}) \qquad (127)$$

is finite, with cardinality

$$| \mathcal{S}_N | = (2(N+1)^3)^{|X|}. \qquad (128)$$

Theorem 2. Eventual periodicity

Every deterministic trajectory

$$S_0, S_1, S_2, \ldots \qquad (129)$$

in $\mathcal{S}_N$ is eventually periodic.

That is, there exist integers

$$t_0 \geq 0, p \geq 1, \qquad (130)$$

such that

$$S_{t+p} = S_t \text{for all } t \geq t_0. \qquad (131)$$

Proof

The trajectory takes values in the finite set $\mathcal{S}_N$. Therefore two states must repeat:

$$S_i = S_j \text{for some } i < j. \qquad (132)$$

Determinism implies identical future evolution from identical states. Hence

$$S_{i+k} = S_{j+k} \text{for all } k \geq 0. \qquad (133)$$

Taking

$$t_0 = i, p = j - i \qquad (134)$$

gives eventual periodicity. ■

Because the trapped sets are monotone,

$$H_t \subseteq H_{t+1}, \qquad (135)$$

a periodic orbit cannot contain continuing horizon growth.

Corollary 4. Horizon stabilization before periodicity

There exists t_H such that

$$H_t = H_{t_H} \text{ for all } t \geq t_H. \qquad (136)$$

Any eventual cycle therefore occurs on a fixed causal graph.

The late-time dynamics separates into

$$\boxed{\text{finite geometric growth} + \text{eventually periodic transport–reaction dynamics.}} \qquad (137)$$

Definition 10. Dispersive trajectory

A trajectory is dispersive when

$$H_t = \emptyset \text{ for all } t. \qquad (138)$$

Definition 11. Collapsing trajectory

A trajectory is collapsing when

$$H_t \neq \emptyset \qquad (139)$$

for some finite t.

The configuration space therefore decomposes into two outcome classes:

$$\mathcal{S}_N = \mathcal{D} \sqcup \mathcal{C}, \qquad (140)$$

where

$$\mathcal{D} = \{S_0 : H_t = \emptyset \text{ for all } t\} \qquad (141)$$

and

$$\mathcal{C} = \{S_0 : H_t \neq \emptyset \text{ for some } t\}. \qquad (142)$$

Definition 12. Finite critical boundary

Let $d$be a metric on $\mathcal{S}_N$. The discrete critical boundary is

$$\partial\mathcal{C} = \left\{S \in \mathcal{S}_N : \inf_{D\in\mathcal{D}} d(S,D) \leq 1 \text{ and } \inf_{C\in\mathcal{C}} d(S,C) \leq 1\right\}. \qquad (143)$$

These are states for which a minimal perturbation can change the outcome between dispersion and trapping.

This is the finite analogue of a threshold hypersurface separating subcritical and supercritical initial data.

Definition 13. Collapse time

For $S_0 \in \mathcal{C}$, define

$$\tau(S_0) = \min\{t : H_t \neq \emptyset\}. \qquad (144)$$

Near the critical boundary, one may ask whether

$$\tau(S_0) \qquad (145)$$

becomes large.

A finite critical-delay law would have the form

$$\tau(S_0) \approx -\gamma_\tau \log d(S_0, \partial\mathcal{C}) + C, \qquad (146)$$

over a suitable range of discrete scales.

Equation (146) is not assumed. It is an experimentally testable property of particular finite models.

Definition 14. Trapped-energy order parameter

Define

$$\mu(S_0) = \lim_{t\to\infty} \frac{E_t^{\text{trap}}}{E_0}. \qquad (147)$$

Because E_t^{trap} is monotone and bounded above by E_0, the limit exists:

$$0 \leq \mu(S_0) \leq 1. \qquad (148)$$

The order parameter distinguishes

$$\mu = 0 \qquad (149)$$

from

$$\mu > 0. \qquad (150)$$

A discrete mass-scaling analogue would be

$$\mu(S_0) \sim d(S_0, \partial\mathcal{C})^{\gamma} \qquad (151)$$

for supercritical states near the boundary.

Again, (151) is a candidate empirical law, not a theorem of the abstract construction.

Proposition 9. Existence of the trapped-energy limit

For persistent absorbing horizons,

$$\lim_{t\to\infty} E_t^{\mathrm{trap}} \qquad (152)$$

exists.

Proof

By Proposition 7,

$$E_{t+1}^{\mathrm{trap}} \geq E_t^{\mathrm{trap}}. \qquad (153)$$

By global conservation,

$$E_t^{\mathrm{trap}} \leq E_0. \qquad (154)$$

A bounded monotone sequence converges. ■

12. Relation to Continuum Mathematical Physics

The finite state variables correspond only schematically to continuum fields.

The matter and antimatter counts approximate moments of distribution functions:

$$m_t(x) \sim \int_{\Omega_x} \int p^0 f_+(t,z,p)\, dP\, dz, \qquad (155)$$

$$\bar{m}_t(x) \sim \int_{\Omega_x} \int p^0 f_-(t,z,p)\, dP\, dz. \qquad (156)$$

The radiation count approximates

$$r_t(x) \sim \int_{\Omega_x} \int p^0 f_\gamma(t,z,p)\, dP\, dz. \qquad (157)$$

The continuum distributions satisfy reactive kinetic equations of the schematic form

$$p^\mu \nabla_\mu f_+ = C_+[f_+, f_-, f_\gamma], \qquad (158)$$

$$p^\mu \nabla_\mu f_- = C_-[f_+, f_-, f_\gamma], \qquad (159)$$

$$p^\mu \nabla_\mu f_\gamma = C_\gamma[f_+, f_-, f_\gamma]. \qquad (160)$$

The collision terms must satisfy local four-momentum conservation:

$$\sum_s \int p^\nu C_s\, dP = 0. \qquad (161)$$

The resulting stress-energy tensor is

$$T^{\mu\nu} = \sum_s \int p^\mu p^\nu f_s\, dP. \qquad (162)$$

Spacetime geometry is then determined by

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \qquad (163)$$

The causal structure of $g_{\mu\nu}$, rather than an additional material field, determines whether future-directed signals can reach infinity.

The finite correspondence is therefore

continuum theory		finite model
f_+, f_-, f_γ	$\leftrightarrow$	$m, \bar{m}, r,$
$C_s[f]$	$\leftrightarrow$	$\mathcal{A},$
$p^\mu \nabla_\mu f_s$	$\leftrightarrow$	$\mathcal{T}_g,$
$T_{\mu\nu}$	$\leftrightarrow$	$e_t(x)$ and $\mathcal{C}_t(x, \rho),$
$g_{\mu\nu}$	$\leftrightarrow$	g_t and $\chi_t,$
trapped surface	$\leftrightarrow$	$H_t.$

(164)

The correspondence is structural, not exact.

12.1 What the finite model preserves

The model preserves the logical implications

$$M + \bar{M} \longrightarrow R, \quad (165)$$

reaction and transport conserve total energy, (166)

concentration, not energy alone, controls trapping, (167)

trapping changes causal accessibility, (168)

and

interior energy is not a new substance. (169)

It also preserves the distinction between global and exterior descriptions:

$$S_t \neq \mathcal{O}_t(S_t). \quad (170)$$

12.2 What the finite model does not derive

The model does not derive:

Lorentz invariance, (171)

Einstein's equations, (172)

a continuum event horizon, (173)

Hawking radiation, (174)

quantum field theory, (175)

or

black-hole thermodynamics. (176)

The threshold rule

$$\mathcal{C} \geq \mathcal{C}_* \qquad (177)$$

is an admitted coarse-graining assumption.

It must not be presented as an exact consequence of general relativity.

12.3 Mathematical status

The construction is therefore best classified as

$$\boxed{\text{a conservative finite reaction–transport system with endogenous causal trapping.}} \qquad (178)$$

Its potentially novel feature is not any individual physical ingredient. The ingredients are familiar:

annihilation,radiative transport,energy conservation,compactness,trapping. (179)

The proposed contribution is their compression into the finite compositional architecture

$$\boxed{F = \mathcal{G} \circ \mathcal{K} \circ \mathcal{T}_g \circ \mathcal{A}.} \qquad (180)$$

This form makes every assumption explicit and every finite trajectory exhaustively auditable.

12.4 Research questions

The model generates concrete questions.

Does the critical boundary

$$\partial\mathcal{C} \qquad (181)$$

possess self-similar structure?

Do collapse times obey a finite scaling law such as

$$\tau \sim -\gamma_\tau \log \mid p - p_* \mid ? \qquad (182)$$

Does trapped energy satisfy

$$E_{\infty}^{\mathrm{trap}} \sim | p - p_* |^{\gamma} ? \qquad (183)$$

How does the operator-splitting defect

$$\Delta_t = \mathcal{T}_g \mathcal{A} - \mathcal{A} \mathcal{T}_g \qquad (184)$$

affect the collapse boundary?

Can distinct microscopic interiors remain exterior-equivalent:

$$S \sim_t S' \qquad (185)$$

while producing identical future exterior histories?

Under what conditions does a finite refinement sequence converge toward a relativistic kinetic–gravitational system?

These questions define the program's mathematical content more sharply than any visual analogy.

12.5 Conclusion

The finite model begins with three energy-bearing sectors:

$$M, \bar{M}, R. \qquad (186)$$

Matter and antimatter annihilate according to

$$M + \bar{M} \longrightarrow R, \qquad (187)$$

without loss of total energy.

Transport redistributes the resulting content:

$$q_{t+1}^{-} = \mathcal{T}_g \mathcal{A}(q_t). \qquad (188)$$

Concentration determines whether the causal graph changes:

$$\mathcal{C} \geq \mathcal{C}_* \Longrightarrow g: 0 \longrightarrow 1. \qquad (189)$$

The trapped state is geometric, not material:

$$\boxed{\text{a horizon is not a fourth substance.}} \qquad (190)$$

The entire construction may therefore be summarized by

$$\boxed{\text{opposition produces radiation;transport produces concentration;concentration may produce trapping.}}$$

The resulting theory is not a replacement for Einstein–Boltzmann dynamics. It is a finite algebraic laboratory in which reaction, conservation, concentration, causal restriction, criticality, and observability can be separated, computed, and verified exactly.

13. References

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